Notes 5.6 ~ Law of Sines and Cosines

Unit 5 - Section 6 ~ Law of Sines and Cosines

Mark lives in Valley Oak Hall at CSUN. He is able to walk in a pretty straight line to Lupin Hall to meet his girlfriend and he measures it to be about 125 yards. His sister lives in Southern Wood. How far is he from his sister and his girlfriend from his sister?

Although trigonometry was discovered and developed using right triangles, it can be applied to other types of triangles as well. An oblique triangle is one which contains no right angles.

Law of Sines is used to solve oblique triangles!

If a, b, and c represent the side lengths opposite \( \angle A \), \( \angle B \), and \( \angle C \) respectively, then

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

Use Law of Sines to solve!
We use Law of Sines if we have 2 angles and one side as shown in the previous example, but we can also use Law of Sines if we have 2 sides and one angle as in this example:

In ΔSTU, \( \angle U = 42^\circ \), ST = 22, and SU = 12. Find the m\( \angle T \) and solve the rest of the Δ.

Area of a Triangle:

Old Formula that will always work:

\[
A = \frac{(b)(h)}{2}
\]

New formula made from Law of Sines:

\[
A = \frac{ab\sin C}{2} \quad \text{or} \quad A = \frac{bc\sin A}{2} \quad \text{or} \quad A = \frac{ac\sin B}{2}
\]

***Notice that each letter a, b, and c are always represented ONCE!

Find the area of ΔDOG.

**This formula works much better if you do not know the height of the triangle!
Law of Cosines

**Law of Cosines** works best when you have all three sides or SAS.

If \( a, b \) and \( c \) represent the sides lengths opposite \( \angle A, \angle B, \) and \( \angle C \) respectively then,

\[
\begin{align*}
    a^2 &= b^2 + c^2 - 2bc \cos A \quad \text{or} \quad b^2 &= a^2 + c^2 - 2ac \cos B \quad \text{or} \quad c^2 &= a^2 + b^2 - 2ab \cos C
\end{align*}
\]

**Once again, all three letters must be represented.**

**Whatever side you are solving for is across from the angle you are using the Cosine of.**

Example: Solve the triangle

\[\text{D} \quad \text{ iso } \quad \text{D} \quad \text{220} \quad \text{E} \quad \text{d} \quad \text{F} \]

Solve the following triangle:

\[\text{A} \quad \text{C} \quad \text{B} \]